Cruising and Hovering Response of a Tail-Stabilized Submersible

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Equations of motion are used as a basis for analyzing the inherent dynamic stability and limit maneuver response of a tail-stabilized submersible in cruising and hovering vertical plane motions. It is shown that this type of vessel can perform adequately in cruising but is subject to highly coupled, unstable hovering motion, especially in stern-to-bow ocean currents. The stability of a submersible with both bow and stern stabilizers, having fore-aft symmetry, also is treated. This type of design has inherent hovering stability, and its symmetry would have a salutory effect on the coupled motions of the vessel.

Nomenclature†

A = frontal area of the main hull, i.e., $A = \pi d^2/4$
B = buoyancy force, i.e., $B = \rho g \forall$, positive
CB = center of buoyancy, origin of body axes‡
CG = center of mass
d = maximum diameter of the main hull
g = acceleration of gravity
I_{yy} = pitch moment of inertia referred to y axis‡
I_{yy}' = typical dimensionless moment of inertia, $I_{yy}' = I_{yy}/[(\rho/2)Al^3]$
K,M,N = hydrodynamic moment components relative to CB in directions of the body axes
l = over-all length of the submersible
MPD = maneuvering propulsive devices or thrusters
m = mass of submersible, W/g
m' = dimensionless mass, $m' = m/[(\rho/2)Al]$
n_x = angular velocity of the longitudinal propeller
p,q,r = angular velocity components of body axes
q' = typical dimensionless angular velocity component, $q' = ql/U$
$\dot{q}$ = typical angular acceleration component
$\dot{q}'$ = typical dimensionless angular acceleration component, $\dot{q}' = \dot{q}l^2/U^2$
t = time
$\mathbf{U}$ = velocity vector of CB relative to fluid
U = magnitude of $\mathbf{U}$
$\mathbf{U}_f$ = velocity vector of fluid relative to earth, constant
$\mathbf{U}_{\mathrm{RES}}$ = resultant of $\mathbf{U}$ and $\mathbf{U}_f$
u,v,w = velocity components of $\mathbf{U}$ in directions of body axes
$\dot{w}$ = typical linear acceleration component
u_f,v_f,w_f = velocity components of $\mathbf{U}_f$ in directions of fluid axes‡
$\forall$ = volume enclosed in external boundary of the submersible
W = weight, mg, of the submersible, including internal water, positive
X,Y,Z = hydrodynamic force components in directions of body axes
X_{w_q}' = typical second derivative of a hydrodynamic force with respect to two motion variables, $X_{wq}' = (\partial^2 X/\partial w \partial q)/[(\rho/2)Al]$
x,y,z = right-handed, rectangular body axes with origin at the CB‡
x_0,y_0,z_0 = right-handed, rectangular axes fixed in the earth‡
x_f,y_f,z_f = right-handed, rectangular fluid axes, parallel to x_0,y_0,z_0, and which move with a constant velocity $\mathbf{U}_f$ relative to earth‡
x_G,y_G,z_G = coordinates of the CG measured in the body axes frame
x_G' = typical dimensionless length, $x_G' = x_G/l$
Z',M' = typical dimensionless hydrodynamic force and moment; $Z' = Z/[(\rho/2)AU^2]$, $M' = M/[(\rho/2)AlU^2]$
Z_q',Z_w',M_δ' = typical dimensionless derivatives of hydrodynamic forces and moments with respect to motion variables§; $Z_q' = (\partial Z/\partial q)/[(\rho/2)AlU]$, $Z_w' = (\partial Z/\partial w)/[(\rho/2)AU]$, $M\delta' = (\partial M/\partial \delta)/[(\rho/2)AlU^2]$
α,β = angles of attack and sideslip, respectively
δ,θ = elevator and pitch angles, respectively
ρ = mass density of water
σ' = dimensionless stability index, $\sigma' = \sigma l/U$

Subscripts

a = appendage quantity
b,s = bow and stern, respectively
c = thruster control quantity
e = equilibrium quantity
h = main hull quantity
m = maximum value of a quantity

Introduction

In the present paper, motion equations are utilized to investigate some of the stability and control problems that may be encountered in the operation of a tail-stabilized submersible that must cruise and hover in an ocean current environment. Most of the examples given to illustrate the behavior of the vessel were obtained from a variation-of-parameter, digital computer study of the vertical plane cruising and hovering limit maneuvers of a 43-ft-long, 8-ft-diam rescue submersible. The results of the work¶ were presented recently in Ref. 1.

Hydrodynamic System

The basic hull form is a streamlined body of revolution with a screw propeller for forward thrust, bow and stern thrusters for hovering pitch, heave, yaw, and sway control, a mercury flow system for roll control, and either a movable ring tail or fixed tail fins with movable rudders and elevators for stabilization and control in cruising operations. In the case of the rescue submersible,[2] a mating bell is near the center and on the bottom of the vessel. This is to attach to the access hatch of a disabled submarine to permit the transfer of 12 survivors at a time to the rescue submersible, and then to another operational submarine. Thus, the need exists for both precise hovering control during mating operations and the cruising maneuver capabilities.

Mathematical Model

Although a mathematical model representing the dynamic behavior of the submersible in six degrees of freedom was developed, this is simplified for treatment of the three-degree

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‡ A sketch of the reference frames is shown in Fig. 1.

§ All hydrodynamic derivatives are evaluated at zero angular velocity, zero acceleration, and zero angles of attack α, side slip β, and elevator δ.
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of-freedom motions in the vertical plane, and the automatic control system is replaced by a series of simple control rules that describe the limit overshoot maneuvers in cruising and hovering flight. This simplified model is complete enough to show the underlying factors that can cause difficulties with the motions of the submersible.

During any given maneuver, it is assumed that the mass m, the pitch moment of inertia I_{yy}, and the fluid current velocity $\mathbf{U}_f$ are constant. It also is assumed that the submersible has geometric symmetry relative to its vertical xz plane and that the main hull is a body of revolution with fore-aft symmetry. The latter assumption permits a relatively simple representation of the hydrodynamic forces and moments in hovering conditions where the angle of attack can vary from 0 to 2π. Extensive experimental data are lacking for these conditions.

The equations of motion in the vertical plane are written using the body axes (x, y, z) and an inertial frame (x_f,y_f,z_f) which is fixed in the fluid. The fluid axes form an inertial frame because $\mathbf{U} = (u_f,0,w_f)$ is constant relative to an inertial frame (x_0, y_0,z_0) fixed in the earth.

The longitudinal propulsive force X_c is used in both cruising and hovering operations, and the vertical bow and stern thrust forces Z_b and Z_s with longitudinal coordinates x_b and $-x_b$, respectively, are used in hovering operations. The movable elevators are used for control in cruising flight.

Under the preceding assumptions, the motion equations are as follows:

$$A_1\dot{u} + A_2\dot{w} + A_3\ddot{\theta} + A_4u\dot{\theta} + A_5|u|\dot{\theta} + A_6w\dot{\theta} + A_7uU + A_8wU + A_9wu + A_{10}\dot{\theta}^2 + A_{11}|\dot{\theta}|\dot{\theta} + A_{12}\sin\theta + A_{13}\cos\theta + A_{14}\delta|u|u = X_c \quad \text{(1a)}$$

$$B_1\dot{u} + B_2\dot{w} + B_3\ddot{\theta} + B_4u\dot{\theta} + B_5|u|\dot{\theta} + B_6w\dot{\theta} + B_7uU + B_8wU + B_9wu + B_{10}\dot{\theta}^2 + B_{11}|\dot{\theta}|\dot{\theta} + B_{12}\sin\theta + B_{13}\cos\theta + B_{14}\delta|u|u = Z_c \quad \text{(1b)}$$

$$C_1\dot{u} + C_2\dot{w} + C_3\ddot{\theta} + C_4u\dot{\theta} + C_5|u|\dot{\theta} + C_6w\dot{\theta} + C_7uU + C_8wU + C_9wu + C_{10}\dot{\theta}^2 + C_{11}|\dot{\theta}|\dot{\theta} + C_{12}\sin\theta + C_{13}\cos\theta + C_{14}\delta|u|u = M_c \quad \text{(1c)}$$

where

$$X_c = a_x|n_x|n_x + b_x|n_x|u \quad \text{(2a)}$$

$$Z_c = Z_s + Z_b \quad \text{(2b)}$$

$$M_c = x_b(Z_s - Z_b) \quad \text{(2c)}$$

$$\dot{x}_0 = u\cos\theta + w\sin\theta + u_f \quad \text{(3a)}$$

$$\dot{z}_0 = -u\sin\theta + w\cos\theta + w_f \quad \text{(3b)}$$

$$x_0 = \int_0^t \frac{dx_0}{d\tau}\,d\tau + x_0(0) \quad \text{(3c)}$$

$$z_0 = \int_0^t \frac{dz_0}{d\tau}\,d\tau + z_0(0) \quad \text{(3d)}$$

$$U = (u^2 + w^2)^{1/2} \quad \text{(3e)}$$

and

$$\begin{aligned}
&A_1 = m - \frac{\rho}{2}AlX_{\dot{u}}' && B_2 = m - \frac{\rho}{2}AlZ_{\dot{w}}' && C_1 = C_6 = mz_G \\
&A_3 = mz_G && B_3 = -mx_G && C_2 = -C_4 = -mx_G \\
&A_6 = m - \frac{\rho}{2}AlX_{wq}' && B_4 = -\left(m + \frac{\rho}{2}AlZ_{qh}'\right) && C_3 = I_{yy} - \frac{\rho}{2}Al^3M_{\dot{q}}' \\
&A_7 = -\frac{\rho}{2}AX_0' && B_5 = -\frac{\rho}{2}AlZ_{qa}' && C_5 = -\frac{\rho}{2}Al^2M_q' \\
&A_{10} = -mx_G && B_8 = -\frac{\rho}{2}AZ_w' && C_8 = -\frac{\rho}{2}AlM_{wa}' \\
&A_{12} = W - B && B_{10} = -mz_G && C_9 = -\frac{\rho}{2}AlM_{wh}' \\
&A_2 = A_4 = A_5 = A_8 = 0 && B_{13} = -(W - B) && C_{11} = -\frac{\rho}{4}Al^3M_{qq}' \\
&A_9 = A_{11} = A_{13} = A_{14} = 0 && B_{14} = -\frac{\rho}{2}AZ_\delta' && C_{12} = Wz_G \\
& && B_1 = B_6 = B_7 = 0 && C_{13} = Wx_G \\
& && B_9 = B_{11} = B_{12} = 0 && C_{14} = -\frac{\rho}{2}AlM_\delta' \\
& && && C_7 = C_{10} = 0
\end{aligned} \quad \text{(4)}$$

Equation (2a) for the propeller thrust X_e was obtained from momentum theory. Although absolute values of velocity and angular velocity components do not ordinarily appear in motion equations for submerged bodies, they are necessary here because in hovering motions there exist physical discontinuities in the derivatives of the forces and moments with respect to these variables when the variable changes sign.

In the case of a main hull with fore-aft symmetry, the only contribution to $B_{11}[= -(\rho/4)Al^2Z_{qq}']$ is due to the tail appendage, and preliminary calculations indicated its effect on the motions to be small. When the bow of the main hull is more full than the stern, as in conventional cases, it is expected that Z_{qq}' will be reduced further.

Some of the hydrodynamic contributions due to the main hull and the appendage are separated in the equations while others are not. For example, the pitch moment due to w is separated because the hull moment is primarily a pure couple which (from potential theory) is a function of the product uw (or $\frac{1}{2}U^2 \sin 2\alpha$), whereas the appendage moment arises from the appendage vertical force, which depends upon the product Uw (or $U^2 \sin\alpha$). In Ref. 1, these theoretical representations are used in regression analyses of experimental force and moment data for a fully appendaged hull in the range $-15° \leq \alpha \leq 90°$, and the results show the data to be well-represented by the theory. It is noted that in cruising operations where $u \approx U$ and α is small, the separation of main hull and appendage effects is of little importance.

Conditions for Cruising, Hovering, and Dynamic Stability

Cruising

When acceptable cruising conditions are possible, the vessel has inherent dynamic stability relative to a level, straight-course equilibrium state (u_e,θ_e,δ_e), where $0 < u_e < u_m$ and $|\delta_e| < \delta_m$, u_m and δ_m being the maximum attainable speed and elevator angle, respectively. An inherently stable vessel can recover from very high degrees of control saturation, whereas the inherently unstable vessel cannot.[3] In a proportional control, the degree of control saturation may be defined as the ratio of the elevator angle δ (or elevator angular velocity $\dot{\delta}$) that would be attained if there were no limit, to its limiting value δ_m (or $\dot{\delta}_m$).

Using small perturbation theory and Eqs. (1), it can be shown[4] that, under constant X_c and zero Z_c and M_c conditions, a submersible is dynamically stable in cruising flight if

$$Z_w'M_q' - (m' + Z_q')M_w' > 0 \tag{5a}$$

and

$$X_0'[Z_w'W'(z_G' - x_G'\theta_e) - M_w'(W' - B')\theta_e] + (W' - B')(Z_\delta'M_w' - Z_w'M_\delta')\delta_e > 0 \tag{5b}$$

where $W' = W/(\rho/2)Au_e^2$ and $B' = B/(\rho/2)Au_e^2$. Inequality (5a) is the hydrodynamic stability criterion, and (5b) reduces to

$$z_G - x_G\theta_e > 0 \tag{6}$$

in the case where $W = B$ (neutral buoyancy), the main operating condition of the submersible. The expressions for θ_e and δ_e are obtained from Eqs. (1) under the conditions that $w_f = 0$, θ_e is small, $W = B$, and $M_\delta' = \frac{1}{2}Z_\delta'$. This gives

$$\theta_e = \frac{Wx_G}{(M_w' - \frac{1}{2}Z_w')(\rho/2)Alu_e^2 - Wz_G} \tag{7}$$

and

$$\delta_e = \frac{-Z_w'Wx_G}{[(M_w' - \frac{1}{2}Z_w')(\rho/2)Alu_e^2 - Wz_G]Z_\delta'} \tag{8}$$

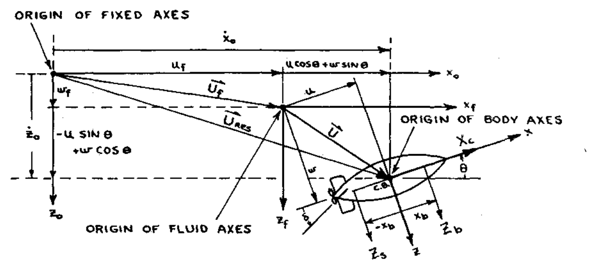

Fig. 1 Sketch showing axes systems and positive directions of various quantities.

Use of (7) in (6) results in

$$z_G - \frac{Wx_G^2}{(M_w' - \frac{1}{2}Z_w')(\rho/2)Alu_e^2 - Wz_G} > 0 \tag{9}$$

as a necessary condition for dynamic stability. For a body with tail fins, the coefficients Z_w', Z_q', M_w', and M_q' can be represented by the relations[5]

$$Z_w' = Z_{wh}' + Z_{wa}' \qquad M_w' = M_{wh}' + \tfrac{1}{2}Z_{wa}' \tag{10a}$$

$$Z_q' = Z_{qh}' + \tfrac{1}{2}Z_{wa}' \qquad M_q' = M_{qh}' + \tfrac{1}{4}Z_{wa}' \tag{10b}$$

Since the quantity

$$M_w' - \tfrac{1}{2}Z_w' = M_{wh}' - \tfrac{1}{2}Z_{wh}' > 0 \tag{11}$$

Eq. (9) shows that

$$z_G > 0 \tag{12}$$

is a necessary condition for dynamic stability of the neutrally buoyant submersible.

The other basis for defining acceptable cruising conditions is that $|\delta_e| < \delta_m$ or, from (8),

$$\left|\frac{Z_w'Wx_G}{[(M_w' - \frac{1}{2}Z_w')(\rho/2)Alu_e^2 - Wz_G]Z_\delta'}\right| < \delta_m \tag{13}$$

The criterion given by (5a) now is applied to the case of a tapered body of revolution with

$$l = 43.25 \text{ ft} \qquad A = 49.3 \text{ ft}^2$$
$$B = 95{,}500 \text{ lb} \qquad I_{yy} = 2.94 \times 10^5 \text{ slug-ft}^2 \tag{14a}$$

The bare hull coefficients are

$$Z_{wh}' = -0.71 \qquad M_{wh}' = 0.98$$
$$Z_{qh}' = -0.14 \qquad M_{qh}' = -0.07 \tag{14b}$$

and the mass coefficient m' for the neutrally buoyant case is 1.40. By substituting Eqs. (10) into (5a) and using the given numerical constants in the resulting expression, it is found that the submersible has neutral hydrodynamic stability when the tail appendage has a normal force rate coefficient $Z_{wa}' = -0.86$. Thus, $-Z_{wa}' > 0.86$ defines one of the limit curves for acceptable cruising performance.

Similarly, the criteria given by (9) and (13) are applied to the hydrodynamically stable conditions defined by

$$Z_{wa}' = Z_\delta' = -1.5 \qquad \delta_m = 0.35 \text{ rad} \tag{15}$$

and Eqs. (14). The cross-hatched areas u_e vs x_G with z_G as parameter shown in Fig. 2 define the region where acceptable cruising performance is possible. The acceptable cruising regions in the u_e, x_G plane are characterized by two roughly triangular areas with the same vertex on the $x_G = 0$ axis and a u_e value there which is always greater than the critical speed** (but approximately equal to it when $z_G \geq 0.03$ ft).

** The critical speed for a given z_G is that for which the denominator of (7) vanishes.

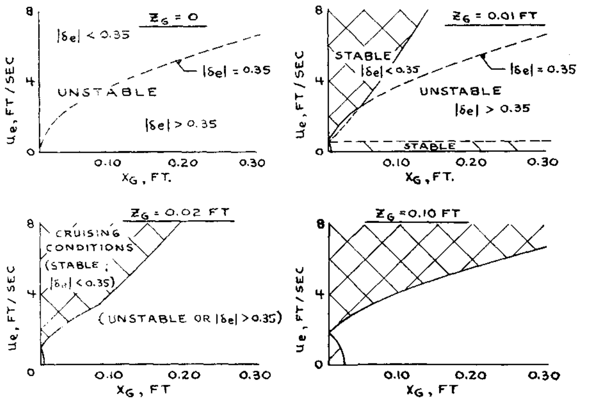

Fig. 2 Regions of speed and c.g. position for stable cruising operations.

The upper area is always much larger than the lower, but both angles at the vertex increase with z_G. The base of the upper area is the maximum speed (assumed here to be $u_m = 8.0$ fps), and the base of the lower area is $u_e = 0$. For the hydrodynamically stable submersible, acceptable cruising conditions are limited by dynamic instability when z_G is small and by the excessively large elevator angles required for equilibrium when z_G is large. In either case, it is necessary to keep x_G small in order to permit cruising at low speeds.

The existence of the cruising regions near $u_e = x_G = 0$ should not be interpreted as being acceptable hovering conditions. The reason for this is that cruising and hovering conditions refer to two completely different equilibrium states. In the cruising condition, no attempt is made to keep the submersible motionless relative to the x_0,y_0,z_0 frame (i.e., $\dot{x}_{0_e} = \dot{z}_{0_e} = 0$), whereas this is precisely the situation in hovering. For example, the case where the submersible drifts with a current $u_f = 1.69$ fps (1 knot), so that its longitudinal speed $u_e = 0$ relative to the fluid is a permissible and acceptable equilibrium cruising condition if $z_G = 0.10$ ft and $x_G = 0.005$ ft (see Fig. 2); however, it is not an admissible hovering condition because $\dot{x}_{0_e} \neq 0$.

Hovering

When acceptable hovering conditions are possible, the vessel has inherent dynamic stability relative to an equilibrium state with $\dot{x}_{0_e} = \dot{z}_{0_e} = 0$, $\theta = \theta_e$, and where $|u_e| < u_m$, $|Z_{s_e}| < Z_m$, and $|Z_{b_e}| < Z_m$, Z_m being the maximum vertical force obtainable with either the bow or stern thruster.

The inherent dynamic stability of the submersible in the hovering equilibrium state

$$\dot{x}_{0_e} = \dot{z}_{0_e} = \theta_e = \dot{\theta}_e = w_e = 0 \qquad u_e = -u_f \tag{16}$$

has been investigated[1] under the conditions that

$$\delta = 0 \qquad Z_s = Z_{s_e} \qquad Z_b = Z_{b_e}$$
$$n_x = n_{x_e} \qquad W - B = 0 \tag{17}$$

Letting

$$u = u_e + \bar{u}, \quad w = \bar{w}, \quad \theta = \bar{\theta}, \text{ etc.} \tag{18}$$

and substituting (16, 17, and 18) into the equations of motion gives

$$n_{x_e} = \left\{\frac{-b_x|u_e| + [(b_x^2 + 4a_xA_7)u_e^2]^{1/2}}{2a_x}\right\}(\mathrm{Sign}\,u_e) \tag{19a}$$

$$Z_{s_e} = C_{13}/2x_b = -Z_{b_e} \tag{19b}$$

for the equilibrium equations, and a system of linearized perturbation equations having a characteristic equation of the form

$$d_0\sigma_i^4 + d_1\sigma_i^3 + d_2\sigma_i^2 + d_3\sigma_i + d_4 = 0 \tag{20}$$

where

$$d_0 = A_1(B_2C_3 - B_3C_2) - C_1A_3B_2$$
$$d_1 = A_1(B_2C_{15} - B_{15}C_2 - B_3C_{17} + B_{17}C_3) - C_1A_3B_{17} - A_{16}(B_3C_2 - B_2C_3)$$
$$d_2 = A_{16}(B_{17}C_3 - B_3C_{17} - B_{15}C_2 + B_2C_{15}) - A_1(B_{15}C_{17} - B_{17}C_{15} - B_2C_{12}) \tag{21}$$
$$d_3 = A_{16}(B_2C_{12} - B_{15}C_{17} + B_{17}C_{15}) + B_{17}A_1C_{12}$$
$$d_4 = A_{16}B_{17}C_{12}$$

with

$$A_{16} = 2|u_e| - b_x|n_{x_e}|$$
$$B_{15} = B_4u_e + B_5|u_e| \qquad B_{17} = B_8|u_e| \tag{22}$$
$$C_{15} = C_4u_e + C_5|u_e| \qquad C_{17} = C_8|u_e| + C_9u_e$$

The submersible is dynamically stable under the prescribe hovering conditions if the real parts of all four σ_i are nega tive. The Routh stability criteria give the necessary an sufficient conditions for this to be true. These are that

$$d_i > 0 \tag{23a}$$

$$d_4 < (d_3/d_1)[d_2 - (d_3d_0/d_1)] \tag{23b}$$

The other bases for defining acceptable hovering conc tions are that $|u_e| < u_m$ (or $|n_{x_e}| < n_{x_m}$), and that each equili rium vertical thrust force be less in magnitude than Z_m, from (19)

$$\left\{\frac{-b_x|u_e| + [(b_x^2 + 4a_xA_7)u_e^2]^{1/2}}{2a_x}\right\} < n_{x_m} \tag{24}$$

and

$$W|x_G|/2x_b < Z_m \tag{24}$$

These hovering criteria now are applied to the submersil with the constants given by (14) and

$$Z_m = 300 \text{ lb} \qquad x_b = 16.0 \text{ ft}$$
$$a_x = 2.46 \text{ slug-ft} \qquad b_x = -6.40 \text{ slugs}$$

The hydrodynamic data include

$$Z_{w_a}' = -1.5 \qquad Z_w' = -2.21$$
$$Z_{q_a}' = M_{w_a}' = -0.75 \qquad M_q' = -0.38$$
$$X_{\dot{u}}' = -0.07 \qquad X_{wq}' = Z_{\dot{w}}' = -1.273$$
$$M_{\dot{q}}' = -0.054 \qquad X_0' = -0.15$$

The values of Z_m and x_b were given, a_x and b_x were de mined using propeller design charts[6] for a three-bladed p peller with diameter of 4.0 ft and pitch-to-diameter ratic 0.50, the coefficients of (26) were obtained using (10) and previously given data (14b), the "added mass and inert coefficients were estimated on the basis of Lamb's "access to inertia" coefficients for prolate spheroids, and the e mate of X_0' is an average value for operating speeds fro to approximately 5 knots. The Schoenherr friction c coefficients, as well as estimates of form drag, were made the main hull, tail appendages, rescue skirt, thruster d and gear, and other exposed equipment to obtain the gi value of X_0'. However, a rough estimate of X_0' is justi because it has been found[1] that reasonably large change its value have little influence on the measures of performa treated herein.

The cross-hatched area of Fig. 3 shows the region in the u_e, z_G plane of acceptable hovering performance defined by the criteria of stability, maximum propeller speed, and maximum vertical thrust forces. The region is closed at the top by the criterion $u_e < 6.95$ fps, which was obtained by arbitrarily assuming a maximum propeller speed n_{xm} of 20 rad/sec in (24a). The notation in Fig. 3 that $|x_G| < 0.10$ ft for $|Z_{se}|$ and $|Z_{be}| < 300$ lb is the result of using the given data in (24b).

The other hovering limit curves in Fig. 3 are the result of applying the stability criteria. It is found that d_0 and d_1 are positive for all u_e, but $d_4 > 0$ only if $z_G > 0$. This latter condition accounts for the $z_G = 0$ limit line in Fig. 3. When $u_e > 0$, $z_G > 0$, and $|x_G| < 0.10$ ft, d_2 and d_3 are positive and condition (23b) is satisfied if

$$\overset{-}{Z_w'}(\overset{-}{M_q'} - m'x_G') - (\overset{+}{m'} + \overset{+}{Z_q'})M_w' > 0 \quad \text{for } u_e > 0 \tag{27a}$$

or, equivalently, if

$$\overset{-}{Z_w'}(\overset{-}{M_q'} - m'x_G') - [\overset{+}{m'} + (\overset{-}{Z_{qh}'} + \overset{-}{Z_{qa}'})] \times (\overset{+}{M_{wh}'} + \overset{-}{M_{wa}'}) > 0 \quad \text{for } u_e > 0 \tag{27b}$$

where the sign of each quantity is shown above its symbol. For the hydrodynamically stable submersible considered, the first product of (27) is positive (stabilizing) and, although the second product gives a negative contribution (destabilizing), its effect is diminished by the stabilizing influence of the tail appendage contributions Z_{qa}' and M_{wa}'. Hence, (27) is satisfied and the neutrally buoyant, tail-stabilized submersible with a small margin of hydrodynamic stability can hover acceptably in large negative (bow-to-stern) ocean currents if $z_G > 0$ and $|x_G|$ is smaller than 0.10 ft.

The sufficient condition for stability analogous to (27) for positive currents (or $u_e < 0$) is that

$$\overset{-}{Z_w'}(\overset{-}{M_q'} + m'x_G') - [\overset{+}{m'} + (\overset{-}{Z_{qh}'} - \overset{-}{Z_{qa}'})] \times (\overset{+}{M_{wh}'} - \overset{-}{M_{wa}'}) > 0 \quad \text{for} \quad u_e < 0 \tag{28}$$

The inequality cannot be satisfied by the tail-stabilized submersible because the tail appendage contribution (Z_{qa}' and M_{wa}') to the second product in (28) causes a destabilizing effect which is greater than that introduced by the body of revolution itself. This places a severe restriction on the hovering capabilities of the tail-stabilized vessel in positive (stern-to-bow) ocean currents and accounts for the lower limit of stability curve (marked d_4) in Fig. 3. This lower limit curve is, to a first-order approximation, defined by

$$u_e > -6(z_G)^{1/2} \qquad (z_G > 0) \tag{29}$$

where z_G is measured in feet and u_e in feet per second. Hence, it is the action of the metacentric pitch moment $-Wz_G \sin\theta$

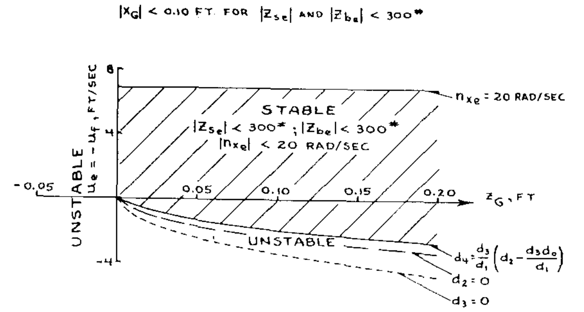

Fig. 3 Regions of current velocity and c.g. position for stable hovering operations of a tail-fin-stabilized submersible.

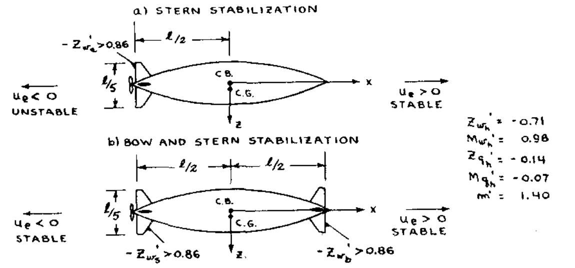

Fig. 4 Comparison of hydrodynamic stability of vessel with a) stern fins only and b) bow and stern fins.

in the vertical-plane motions of the vessel which accounts for the presence of a small stable hovering region when $u_e < 0$. In pure horizontal-plane motions where there is no stabilizing metacentric yaw moment present, the tail-stabilized submersible is inherently unstable in hovering equilibrium conditions with $u_e < 0$. It also is noted that in the numerical applications of (23), the terms containing x_G are found to have a very small contribution when $|x_G| < 0.10$ ft. Hence, the only restriction on x_G shown in Fig. 3 is that given by (24b).

One way in which the hovering stability problem can be eliminated is depicted in Fig. 4. Bow fins are added to the stern-finned submersible to obtain a vessel with fore-and-aft symmetry which is hydrodynamically stable in both directions of motion.

This can be demonstrated by first writing the hydrodynamic rate coefficients for the bow and stern fin case in a manner analogous to (10), using $-\frac{1}{2}Z_{wb}'$ for the bow fin contributions to M_w' and Z_q', and $\frac{1}{4}Z_{wb}'$ for its contribution to M_q'. Then, if it is assumed that both fins are hydrodynamically equivalent, so that $Z_{wb}' = Z_{ws}' = Z_{w1}'$, the appendages have the following net effects on the rate coefficients:

$$Z_{wa}' = 2Z_{w1}' \qquad Z_{qa}' = M_{wa}' = 0 \qquad M_{qa}' = \tfrac{1}{2}Z_{w1}' \tag{30}$$

Use of these relations in (27) and (28) with $x_G = 0$ shows that the stability criteria for $u_e > 0$ and $u_e < 0$ reduce identically to the same one, namely

$$Z_w'M_q' - (m' + Z_{qh}')M_{wh}' > 0 \tag{31}$$

or

$$Z_{w1}'^2 + (\tfrac{1}{2}Z_{wh}' + 2M_{qh}')Z_{w1}' + [Z_{wh}'M_{qh}' - (m' + Z_{qh}')]M_{wh}' > 0 \tag{32}$$

Substitution of the bare hull hydrodynamic coefficients given by (14b) and the mass coefficient $m' = 1.40$ into (32) and solving for Z_{w1}' shows that if $-Z_{w1}' > 0.86$ the bow- and stern-finned vessel will be hydrodynamically stable in both positive and negative ocean currents. It is noted that the value of Z_{wa}' for neutral hydrodynamic stability of the submersible with only a stern appendage also is -0.86. This type of result occurs generally because a stern appendage alone, regardless of size, cannot stabilize the vessel when u_e is negative and the bow appendage alone, regardless of size, cannot stabilize it when u_e is positive.[5]

Limit-Maneuver Response Characteristics

The adequacy of stern stabilization alone in a submersible that must both cruise and hover in ocean currents is investigated further by resort to limit-maneuver response calculations for the tail-stabilized submersible. Although Eqs. (1–4) were used in digital machine computations of one cruising maneuver and three hovering maneuvers,[1] for brevity only one type of the latter class is considered herein. Similarly, the parametric studies covered in Ref. 1 were extensive,

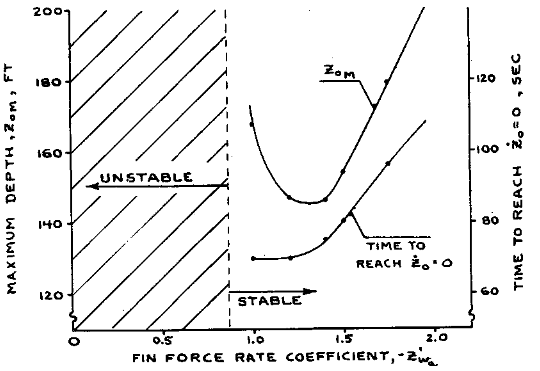

Fig. 5 Effect of Z_{w_a}' on maximum depth and time to complete cruising speed limit maneuver.

but the present discussions deal only with those aspects relating to the use of tail stabilization.

Cruising Limit Maneuvers

In a typical cruising pitch overshoot maneuver, the vessel is initially in straight, level, equilibrium flight with $u_e = 5.07$ fps. The water current velocity components $u_f = w_f = 0$, $Z_\delta' = 2M_\delta' = -0.50$, $Z_{w_a}' = -1.5$, and $x_G = z_G = 0.01$ ft. The other data (where applicable) are the same as given previously in (14, 25, and 26). The elevator then is deflected at the maximum rate $\dot{\delta}_m = 0.20$ rad/sec until it reaches the maximum angle $\delta_m = 0.35$ rad and held there until the time t_1 when the vessel reaches the prescribed execute pitch angle $\theta_1 = -0.80$ rad. At time t_1, the elevator is deflected at the rate $-\dot{\delta}_m$ until it reaches the angle $-\delta_m$ and is held there until the time t_2 when the depth velocity $\dot{z}_0$ is zero. The run is stopped when $t = t_2$. The run also is stopped at any time when $|\theta| > 1.57$ rad or $t > 100$ sec. Some of the measures of performance are θ_e, δ_e, t_1, z_{01}, z_{0m}, θ_m, and t_2.

Figure 5 shows how the maximum depth z_{0m} and the time t_2 vary with Z_{w_a}' for the previously given values of the other parameters. There exists a value $Z_{w_a}' \approx -1.3$ for minimum z_{0m} and nearly minimum t_2. The submersible with $0.86 < |Z_{w_a}'| < 1.3$ may also respond rapidly but, because of its marginal stability, requires larger depths than in the optimum case to complete the maneuver. A submersible with $|Z_{w_a}'| < 0.86$ cannot complete the maneuver because the pitch angle θ exceeds the limit of 1.57 rad. For example, when $Z_{w_a}' = -0.70$, θ exceeds 1.57 rad after only 26.0 sec and while the depth velocity is still increasing. This confirms the result of the previous stability analysis. Lastly, when $|Z_{w_a}'| > 1.3$, the vessel has an excessive amount of hydrodynamic stability and requires greater depth and longer time to complete the maneuver than in the optimum case. Since an automatic control with elevator deflected in proportion to an error in $\dot{\theta}$ would (in effect) add hydrodynamic stability to the system, it would be preferable to choose a tail fin with $0.86 < |Z_{w_a}'| < 1.3$ than one with $|Z_{w_a}'| > 1.3$.

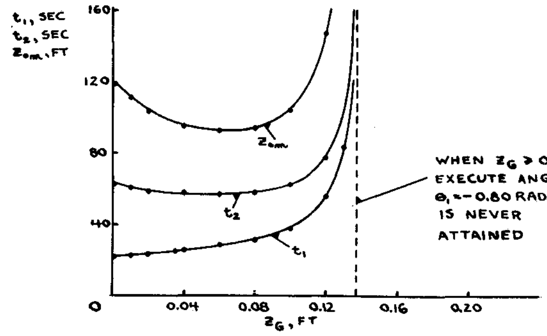

Fig. 7 Effect of vertical c.g. position on cruising pitch maneuver response for otherwise standard run conditions.

Figure 6 shows the effect of maximum elevator force coefficient $|Z_\delta'|\cdot\delta_m$ on cruising maneuver response. The standard condition with $Z_{w_a}' = -1.5$ was used to obtain the fairly good collapse of the z_{0m} and t_2 data. These results show that both the depth and time required to complete the maneuver decrease as the maximum elevator force coefficient increases.

Figure 7 shows how t_1, t_2, and z_{0m} vary with metacentric height z_G for conditions with $\delta_m = 0.175$ rad and $Z_\delta' = Z_{w_a}' = -1.5$. The total depth z_{0m} and time t_2 are minimized when $z_G \approx 0.065$ ft. The submersible with $0 < z_G < 0.065$ ft also responds rapidly, but its marginal stability causes larger depth excursions than in the optimum case. All three measures of performance increase from the values at $z_G = 0.065$ ft to infinite values at $z_G = 0.137$ ft. It can be shown analytically[1] that the submersible with $Z_\delta' = -1.50$ and δ fixed at $\delta_m = 0.175$ rad attains a steady, downward, straight course condition with $|\theta| < 0.80$ rad for $z_G \geq 0.137$ ft. It is for this reason that the execute angle of $\theta_1 = -0.80$ rad is never attained in the cruising maneuver computations when $z_G > 0.137$ ft. Hence $z_G = 0.137$ ft is a limiting value for the standard maneuver conditions.

However, values of z_G which are excessive for cruising operations might be desirable for hovering maneuvers. Therefore, the variation of the limiting values of z_G with both θ_1 and $Z_\delta'\delta$ was investigated[1] using the otherwise standard conditions given previously. Figure 8 shows this variation with θ_1, and indicates that the limiting value of z_G decreases with increasing $|\theta_1|$, approximately like $-0.1/\sin\theta_1$. If a value of $z_G < 0.09$ ft is chosen for cruising operations, all pitch angles will be attainable for the submersible with standard conditions.

Figure 9 shows how this limiting value of z_G can be increased by increasing the maximum elevator force coefficient $|Z_\delta'|\cdot\delta_m$. It is a plot of the limiting value of z_G vs $|Z_\delta'|\cdot\delta_m$ for $\theta_1 = -0.80$ rad and otherwise standard conditions. The limit value of z_G increases linearly with $|Z_\delta'|\cdot\delta_m$, and has a

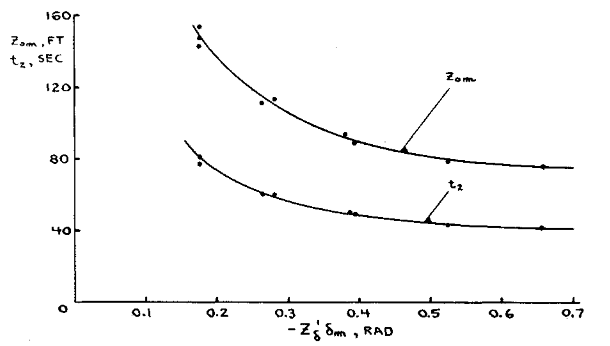

Fig. 6 Effect of the product $Z_\delta'\delta_m$ on cruising pitch overshoot maneuver for otherwise standard run conditions.

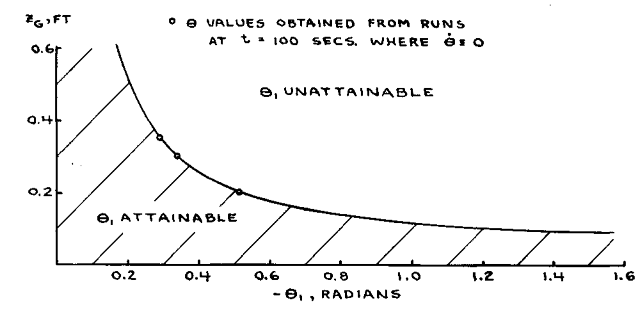

Fig. 8 Limit values of vertical c.g. position above which execute pitch angle cannot be attained for otherwise standard cruising pitch maneuver conditions.

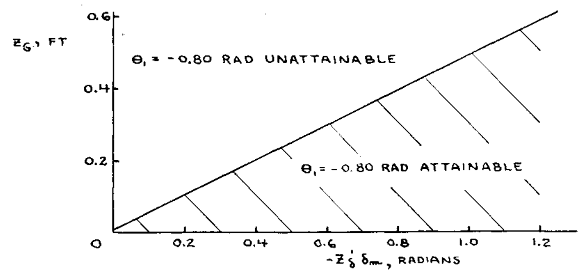

Fig. 9 Limit values of z_G for $\theta_1 = 0.8$ rad as a function of maximum elevator force coefficient for otherwise standard cruising pitch maneuver conditions.

value of about 0.30 ft when $|Z_\delta'\delta_m| = 0.60$. If $|Z_\delta'\delta_m| = 0.60$, it can be shown[1] that all values of θ_1 are attainable when $z_G < 0.209$ ft.

Therefore, it has been shown that, for a given value of $Z_\delta' \cdot \delta_m$, there exists a tail-appendage force rate coefficient Z_{w_a}' and a metacentric height z_G which yield optimum cruising overshoot maneuver response. However, the optimum value of z_G for cruising is small, and it may be desirable to increase z_G for hovering operations. This can be done, without restricting the attainable pitch angles of the submersible in cruising, by increasing the value of the maximum elevator force coefficient $|Z_\delta'| \cdot \delta_m$.

Hovering Limit Maneuvers

In a typical heave overshoot maneuver with a bow-to-stern current, the tail-stabilized submersible operates under otherwise standard conditions of

$$\begin{aligned} &Z_{w_a}' = -1.5 \qquad \delta = 0 \qquad M_{qq}' = -0.42\dagger\dagger \\ &Z_m = 300 \text{ lb} \qquad \dot{Z}_m = 100 \text{ lb/sec} \\ &W = B = 95{,}500 \text{ lb} \qquad x_G = z_G = 0.01 \text{ ft} \\ &w_f = 0 \qquad z_{0_1} = 20 \text{ ft} \end{aligned} \tag{33}$$

With the vessel initially motionless with zero pitch angle relative to the x_0,y_0,z_0 inertial frame, the bow and stern vertical thrust forces Z_b and Z_s are increased from their respective equilibrium values $Z_{b_e} = -29.84$ lb and $Z_{s_e} = 29.84$ lb at the maximum rate $\dot{Z}_m = 100$ lb/sec until the stern vertical thrust force reaches the maximum value $Z_m = 300$ lb and the bow vertical thrust force reaches the value of 240.32 lb. These forces are maintained until time t_1 when the prescribed execute depth $z_{0_1} = 20$ ft is reached. At time t_1, the Z_b and Z_s forces begin to reverse at the rate $-\dot{Z}_m$ until Z_b reaches the maximum negative value of -300 lb and $Z_s = -240.32$ lb. These vertical thrust forces are maintained until the time t_2 when the depth z_{0_1} of 20 ft again is attained, and the run is stopped. A run also is stopped if $|z_0| > 100$ ft, $|\theta| > 1.57$ rad, or $t > 200$ sec. In a completed hovering heave maneuver, there is a time in the interval (t_1,t_2) where $\dot{z}_0 = 0$. It is noted that the control moment M_c and the longitudinal propeller speed n_x have the equilibrium values $M_{c_e} = x_b(Z_{s_e} - Z_{b_e})$ and n_{x_e} for all time $(0,t_2)$ in a hovering heave maneuver. Hence, any net unbalanced pitch moments or longitudinal forces are due entirely to transient inertial and hydrodynamic effects. Some of the measures of performance are Z_{b_e}, Z_{s_e}, t_1, θ_1, x_{0_1}, θ_m, x_{0m}, z_{0m}, and t_2.

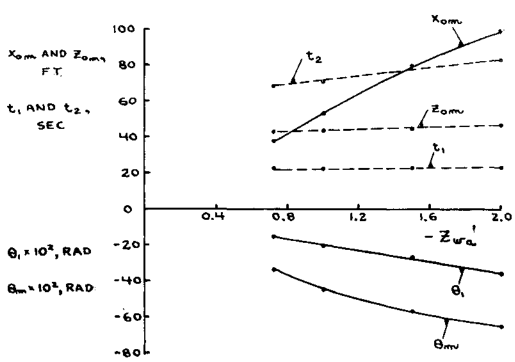

Fig. 10 Effect of tail-fin force rate coefficient on hovering heave overshoot maneuver for otherwise standard run conditions.

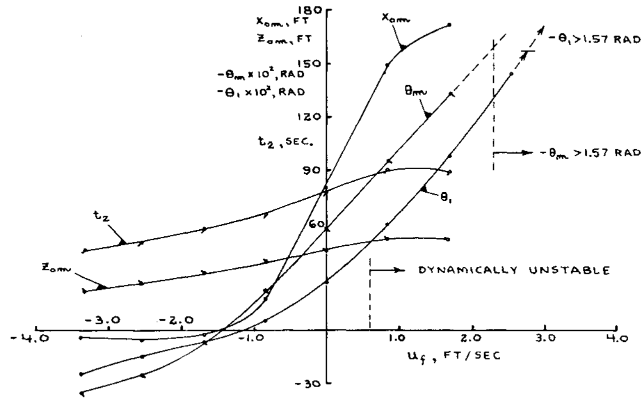

Fig. 11 Effect of longitudinal current velocity on hovering heave maneuver for otherwise standard run conditions.

Figure 10 shows the effect of Z_{w_a}' on six measures of performance in the hovering maneuver for otherwise standard conditions with $u_c = 0$ (i.e., $u_f = 0$). Although the vessel is marginally stable for all Z_{w_a}' when $u_e = 0$, the asymmetry of the submersible due to the presence of the tail appendage is manifested in the increasingly large heave-to-surge and heave-to-pitch coupling effects that are obtained as $|Z_{w_a}'|$ is increased. For example, when $Z_{w_a}' = -1.5$, the vessel attains a pitch angle of almost -33 deg and moves forward a distance of 80 ft before the heave maneuver is completed. This occurs without applying any unbalanced pitch moments M_c or longitudinal forces X_c to the system. The source of this behavior is the nose-down hydrodynamic pitch moment due to the tail appendage $(-C_8wU)$ which acts when a pure downward force Z_c is applied to the vessel. The tail-stabilized submersible also is characterized by its slow response and large depth overshoot in hovering heave maneuvers with $u_e = 0$.

Figure 11 demonstrates the strongly adverse effects that positive longitudinal current velocities u_f have on the hovering heave maneuver response of the tail-stabilized submersible for otherwise standard conditions. These results confirm the trends predicted by the analysis of dynamic stability. For a positive current velocity of 1 knot, the vessel attains a pitch angle θ and longitudinal displacement of almost 80 deg and 4 hull lengths, respectively, before the maneuver is completed. At positive currents slightly higher than 1 knot, the maneuver cannot be completed at all because the pitch angle exceeds the limit value. On the other hand, for a negative current velocity of 1 knot, both the response time t_2 and the heave overshoot are significantly reduced and the coupling effects are negligible in comparison with those obtained for the positive 1-knot current.

It is noted that attempts to improve hovering response in positive longitudinal currents by large reductions in tail-appendage size were unsuccessful. These reductions did not improve heave response significantly for $u_f > 0$, and also had large detrimental effects on performance for $u_f < 0$.

Because the pitch and surge motions of the tail-stabilized submersible are so sensitive to the action of a pure vertical force Z_c, some amelioration of these coupled motions can be

†† M_{qq}' was determined analytically[1] using strip theory. The resulting expression is $M_{qq}' = \frac{1}{16}Z_{wh}' + \frac{1}{4}Z_{wa}'$.

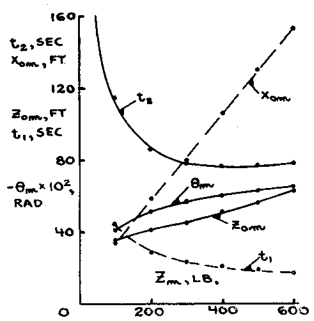

Fig. 12 Effect of maximum vertical MPD force on hovering heave maneuver for otherwise standard run conditions.

obtained by reducing the magnitude of Z_m. Figure 12 shows this effect for otherwise standard conditions with $u_e = 0$. Decreasing Z_m also reduces the heave overshoot and causes large increases in response time. Therefore, if the slower response can be accepted, small vertical control forces (50 or 100 lb) could be used to move the vessel vertically. This will permit the hydrodynamic pitch moments arising from the vertical motion to be counteracted by a pure control moment M_c.

It is noteworthy that reducing the rate of application $\dot{Z}_m$ of the vertical thrust force does not have the same mitigating effects on hovering heave maneuver response as does reducing Z_m. Figure 13 shows that reducing $\dot{Z}_m$ increases the coupling effects as well as the time of response and depth overshoot. However, any improvement in performance to be gained by increasing $\dot{Z}_m$ will be small for $\dot{Z}_m > 100$ lb/sec.

The only other method found to improve the vertical-plane, hovering limit-maneuver response of the tail-stabilized submersible is to increase the metacentric height z_G. Figure 14 shows this for otherwise standard conditions with $u_e = 0$. Increasing z_G from 0.01 to 0.10 ft decreases x_{0m} from 79.5 to 21.5 ft, θ_m from -0.57 to -0.33 rad, t_2 from 78 to 61 sec, and z_{0m} from 44.9 to 38.2 ft. Further increase in z_G has little effect on response time and depth overshoot, but continues to reduce the pitch and surge coupling effects.

Concluding Remarks

Analyses of inherent dynamic stability and limit-maneuver response have shown that the tail-stabilized submersible can perform adequately in cruising flight, but is subject to highly-coupled, unstable hovering motions, especially in stern-to-bow ocean currents. Two methods have been suggested for improving its performance, but these are restricted in applicability. The first is to reduce pitch and surge coupling effects by limiting vertical velocity through application of only small unbalanced vertical control forces and the use

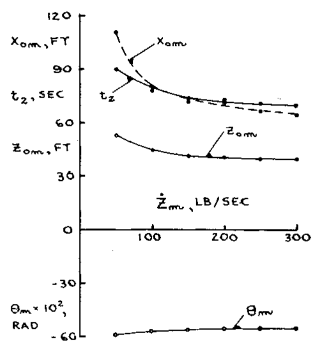

Fig. 13 Effect of maximum MPD vertical force rate on hovering heave maneuver for otherwise standard run conditions.

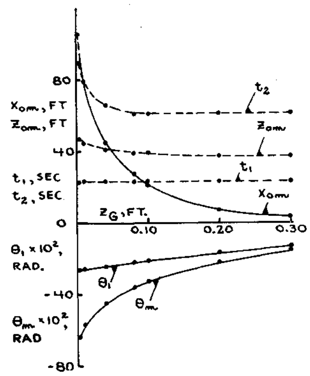

Fig. 14 Effect of metacentric height on hovering heave maneuver response for otherwise standard run conditions.

of corrective pitch control moments. This method will be difficult to apply in the vertical plane because of the extreme sensitivity of the hovering control and motions of the vehicle to small positive or negative buoyancy forces and vertical currents. It also will be difficult to apply in the horizontal plane where large lateral currents can act. In these cases, control saturation can cause loss of hovering control of the inherently unstable submersible.

The second method is to increase the inherent vertical-plane hovering stability of the vessel by increasing its metacentric height. The larger metacentric pitch moment acts as an inherent pitch angle control, and a reduction in cruising capability can be avoided by increasing the maximum elevator force coefficient $|Z_\delta'| \cdot \delta_m$. However, in horizontal-plane motions where metacentric yaw moments are absent, large sway-to-yaw coupling effects cannot be avoided with the tail-stabilized vessel.

Another approach to the design of a submersible that must cruise and hover also has been considered. If all appendages are confined to lie within the maximum hull radius, this approach is to use bow and stern stabilizers which are hydrodynamically equivalent. A design of this type is inherently stable with respect to all equilibrium hovering conditions, and its fore-aft symmetry eliminates one source of hydrodynamic coupling that is present in the tail-stabilized submersible. The elevators can be part of the stern stabilizers for better turning and diving effectiveness in cruising. The bow stabilizers are not expected to have any detrimental influence on cruising performance. In particular, they do not affect cruising stability, and may have a beneficial effect on turning ability.

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